1. The number of tornadoes per year to hit a particular town follows a Poisson distribution with mean λ. A weatherman claims that due to climate changes the mean number of tornadoes per year has decreased. He records the number of tornadoes x to hit the town last year.

To test the hypotheses H_0: $\lambda = 7$ and H_1: $\lambda < 7$, a critical region of $x \leq 3$ is used.

(a) Find, in terms λ the power function of this test.

(3)

(b) Find the size of this test.

(2)

(c) Find the probability of a Type II error when $\lambda = 4$.

(2)
(Total 7 marks)

2. The number of accidents that occur at a crossroads has a mean of 3 per month. In order to improve the flow of traffic the priority given to traffic is changed. Colin believes that since the change in priority the number of accidents has increased. He tests his belief by recording the number of accidents x in the month following the change. Colin sets up the hypotheses H_0: $\lambda = 3$ and H_1: $\lambda > 3$, where λ is the mean number of accidents per month, and rejects the null hypothesis if $x > 4$.

(a) Find the size of the test.

(3)

The table gives the values of the power function of the test to two decimal places.

λ	4	5	6	1
Power	r	0.56	s	0.83

(b) Calculate the value of r and the value of s.

(2)

(c) Comment on the suitability of the test when $\lambda = 4$.

(1)
(Total 6 marks)

3. (a) Define

(i) a Type I error,

(ii) a Type II error.

(2)

A small aviary, that leaves the eggs with the parent birds, rears chicks at an average rate of 5 per year. In order to increase the number of chicks reared per year it is decided to remove the eggs from the aviary as soon as they are laid and put them in an incubator. At the end of the first year of using an incubator 7 chicks had been successfully reared.

(b) Assuming that the number of chicks reared per year follows a Poisson distribution test, at the 5% significance level, whether or not there is evidence of an increase in the number of chicks reared per year. State your hypotheses clearly.

(4)

(c) Calculate the probability of the Type I error for this test.

(3)

(d) Given that the true average number of chicks reared per year when the eggs are hatched in an incubator is 8, calculate the probability of a Type II error.

(2)
(Total 11 marks)

1.	(a)	Power = P($X \le 3/\lambda$)		M1	
		$= e^{-\lambda} + e^{-\lambda}\lambda + \dfrac{e^{-\lambda}\lambda^2}{2} + \dfrac{e^{-\lambda}\lambda^3}{6}$		A1	
		$= \dfrac{e^{-\lambda}}{6}(6 + 6\lambda + 3\lambda^2 + \lambda^3)$		A1	3
	(b)	CR is $X \le 3$		M1	
		Size = P[$X \le 3 / \lambda = 7$]			
		= 0.0818		A1	2
	(c)	P(Type II errors) = 1 – power			
		$= 1 - \dfrac{e^{-4}}{6}(6 + 6\times 4 + 3\times 4^2 + 4^3)$		M1	
		= 0.5665...		A1	2
					[7]
2.	(a)	Size of text = P(X > 4 | λ = 3)		M1	
		= 1 – P(X ≤ 4 | λ = 3) = 1 – 0.8153		A1	
		= 0.1847	awrt 0.185	A1	3
	(b)	r = 1 – 0.6288 = 0.3712 = 0.37 (2dp)		B1	
		s = 1 – 0.2851 = 0.7149 = 0.71 (2dp)		B1	2
	(c)	When λ = 4, Power = 0.27 < 0.5			
		Probability of coming to correct conclusion is less than probability of coming to wrong conclusion.			
		Not suitable.		B1	1
3.	(a)	(i) Type I – H_0 rejected when it is true		B1	
		Type II – H_0 is accepted when it is false		B1	2
	(b)	H_0: $\lambda = 5$, H_1: $\lambda > 5$	both	B1	
		P(($X \ge 7 \mid \lambda = 5$) = 1 – 0.7622 = 0.2378 >0.05		M1 A1	
		(**OR** P($X \ge 9$) = 0.0681, P($X \ge 10$) = 0.0318, CV = 10, 7 not in CR		M1 A1	
		probabs & 10, dnr			
		No evidence of an increase in the number of chicks reared per year.		A1	4
		Context			
	(c)	P($X \ge c \mid \lambda = 5$) < 0.05		M1	
		P($X \ge 9$) = 0.0681, P($X \ge 10$) = 0.0318, c = 10		M1	
		may be seen in (b)			

	P(Type I Error) = 0.0318	A1	3
(d)	$P(X \le 9) \mid \lambda = 8) = 0.7166$	M1 A1	
	(OR if $c = 9$ in (d), $P(X \le 8 \mid \lambda = 8) = 0.5925$	M1 A1	2

[11]

1. The vast majority of candidates realized that Poisson distributions should be used in this question. The concepts of size and power were generally well understood, but a minority of candidates did not use lambda in part (a) or worked out $P(X > 3 / \lambda)$. Parts (b) and (c) were generally correct.

2. Parts (a) and (b) were attempted well, but a correct answer to part (c) was rare.

3. The definitions were remembered well for part (a) and the test in part (b) was often correct. However parts (c) and (d) discriminated between candidates, with many incorrect probabilities. Some solutions interpreted the critical region to be above 0.05 and so lost an accuracy mark in part (c), others struggled with the regions required and simply guessed at the probabilities.